# Application of a New Multivariable Model-Following Method to Decoupled Flight Control

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**A new model-following controller for multivariable, linear, time-invariant systems is described and applied to the decoupled longitudinal control of a control-configured-vehicle-type aircraft. The controller is composed of an input dynamics compensator and a state feedback block. This approach enables decoupling and control of systems that cannot be done by state feedback alone. The key concept is that of system augmentation by utilizing a unimodular matrix to assure the nonsingularity of a control matrix. Two methods of generating the control input are described. In the first, the control input is synthesized by explicitly using the plant state variables. In the second, the input and output of the plant are used.**

## Introduction

THE design of flight control systems for a high-performance aircraft often requires that the aircraft be considered as a multi-input multi-output system. However, the coupling between the plant inputs and outputs adversely affects the flying qualities of the aircraft. For this reason the system must be decoupled for good control. This is especially true for control-configured-vehicle (CCV)-type aircraft which employ such techniques as direct force, fuselage aiming, etc.

The necessary and sufficient condition for system decoupling using state feedback was first given in Ref. 1. This condition requires that a certain matrix (the control matrix, in this paper) that depends upon the system's parameters be nonsingular. A solution is presented in Ref. 2 for the case where this condition is not satisfied. However, this method is not systematic and gives a solution only to a class of systems. A more general solution can be obtained using the exact model-matching concept,[3] which makes the transfer function matrix of the plant controller combination approach that of the model.

In this paper, the design of a model-following controller based upon the exact model-matching concept for multivariable linear continuous time-invariant systems is presented. Its application to the decoupled flight control of the longitudinal motion for CCV-type aircraft is also considered. A brief description is given first of a multivariable model-following control problem. The proposed scheme is then presented for two cases: 1) where the state variables are used explicitly to generate the control input, and 2) where only the plant inputs and outputs are used. Two examples of the CCV flight controllers and numerical simulations are given which verify that the proposed methods are effective.

Received Aug. 5, 1982; revision received Nov. 16, 1984. 

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## Problem Statement

Consider a possibly unstable controllable and observable plant described by the following equations:

$$\dot{x}(t) = Ax(t) + Bu_P(t), \qquad x(0) = x_0 \tag{1a}$$

$$y(t) = Cx(t) \tag{1b}$$

where $x(t) \epsilon R^n$, $u_P(t)$ and $y(t) \epsilon R^p$.

The transfer function matrix (TFM) of Eqs. (1) is written as

$$H(s) = C(sI - A)^{-1}B \tag{2}$$

where the rank of $C_{\text{adj}}(sI-A)B$ is $p$ (full rank) and all of the plant zeros are located on the left-hand side of the $s$ plane. (In Eq. (2), $s$ denotes both Laplace and differential operators depending upon the context.) A reference model which has the desired characteristics is represented by

$$\dot{x}_M(t) = A_M x_M(t) + B_M u_M(t), \qquad x_M(0) = x_{M0} \tag{3a}$$

$$y_M(t) = C_M x_M(t) \tag{3b}$$

where $x_M(t) \epsilon R^{\bar{n}}$, $u_M(t)$ and $y_M(t) \epsilon R^p$. The corresponding TFM is

$$H_M(s) = C_M(sI - A_M)^{-1}B_M \tag{4}$$

where $\det(sI - A_M)$ is a stable polynomial. In order that pure differentiation be avoided in the design, assume that $\xi_P \xi_M^{-1}$ is proper, where $\xi_P$ and $\xi_M$ are the interactor matrices[4] of the plant and model, respectively.

The problem of concern here is to design a controller that makes the plant outputs follow the model outputs in the sense of exact model matching. If the model TFM is chosen to be diagonal with desired pole-zero locations, the TFM of the plant/controller combination is forced to be decoupled and to have the characteristics specified by the model. Therefore, this model-following concept includes decoupling of the system, which implies that this design can be used for nondecoupled control as well, depending upon the selection of the model TFM.

## Model-Following Controller Design

### Using State Variables

The successive differentiation of each output $y_j(t)$ of Eq. (1b) and substitution of Eq. (1a) into the resulting equation gives

$$y_j^{(k)}(t) = C_j A^k x(t), \qquad (k=0,1,\ldots,m_j-1) \tag{5a}$$

$$y_j^{(m_j)}(t) = C_j A^{m_j} x(t) + C_j A^{m_j-1} B u_P(t) \tag{5b}$$

where $y_j(t)$ stands for the $j$th element of $y(t)$, $y_j^{(k)}(t)$ for the $k$th time derivative of $y_j(t)$, and $m_j$ for the $j$th row relative degree, in which the relative degree means the difference order of denominator and numerator, of the plant TFM given by Eq. (2). Similarly, the following relations can be obtained for the model:

$$y_{Mj}^{(k)}(t) = C_{Mj} A_M^k x_M(t), \qquad (k=0,1,\ldots,m_j-1) \tag{6a}$$

$$y_{Mj}^{(m_j)}(t) = C_{Mj} A_M^{m_j} x_M(t) + C_{Mj} A_M^{m_j-1} B_M u_M(t) \tag{6b}$$

When $\xi_P \xi_M^{-1}$ is strictly proper, then,

$$C_{Mj} A_M^{m_j-1} B_M = 0 \tag{7}$$

Selecting the coefficients $f_k^j$ $(k=1,2,\ldots m_j,\ j=1,2,\ldots,p)$ such that the polynomial

$$f^j(s) = s^{m_j} + f_1^j s^{m_j-1} + \ldots + f_{m_j}^j \tag{8}$$

is stable, Eqs. (5) and (6) can be rewritten as

$$y_j^{(m_j)}(t) + f_1^j y_j^{(m_j-1)}(t) + \ldots + f_{m_j}^j y_j(t) = (C_j A^{m_j} + f_1^j C_j A^{m_j-1} + \ldots + f_{m_j}^j C_j) x(t) + C_j A^{m_j-1} B u_P(t) \tag{9a}$$

$$y_{Mj}^{(m_j)}(t) + f_1^j y_{Mj}^{(m_j-1)}(t) + \ldots + f_{m_j}^j y_{Mj}(t) = (C_{Mj} A_M^{m_j} + f_1^j C_{Mj} A_M^{m_j-1} + \ldots + f_{m_j}^j C_{Mj}) x_M(t) + C_{Mj} A_M^{m_j-1} B_M u_M(t) \tag{9b}$$

From the above, the following error equation is obtained

$$\Phi(s) e(t) = F x(t) + G u_P(t) - F_M x_M(t) - G_M u_M(t) \tag{10}$$

where

$$e_j(t) = y_j(t) - y_{Mj}(t), \qquad (j=1,2,\ldots,p) \tag{11a}$$

$$e^T(t) = [e_1(t), e_2(t), \ldots, e_p(t)] \tag{11b}$$

$$\Phi(s) = \mathrm{diag}[f^j(s)], \qquad (j=1,2,\ldots,p) \tag{11c}$$

$$G = [G_j], \qquad (j=1,2,\ldots,p); \qquad G_j = C_j A^{m_j-1} B \tag{11d}$$

$$F = [F_j], \qquad (j=1,2,\ldots,p)$$

$$F_j = C_j A^{m_j} + f_1 C_j A^{m_j-1} + f_{mj} C_j \tag{11e}$$

In the above, $G$ is the control matrix, and $G_M$ and $F_M$ are defined for the model in the same way as $G$ and $F$ for the plant.

*Case I: Nonsingular Control Matrix*

Define a control input such that the right-hand side of Eq. (10) is equal to zero; that is,

$$u_P(t) = G^{-1}[-F x(t) + q(t)] \tag{12a}$$

where

$$q(t) = F_M x_M(t) + G_M u_M(t) \tag{12b}$$

As $\Phi(s)$ in Eq. (10) was chosen to be stable, the output error tends to zero. In other words, the plant output follows the model output.

Let us now show that the control input [Eqs. (12)] leads to exact model matching. The output $y(s)$ can be expressed as[5]

$$y(s) = H(s)[G + F(sI-A)^{-1}B]^{-1} q(s) = H(s)[\Phi(s)H(s)]^{-1} q(s) = \Phi^{-1}(s) q(s) \tag{13}$$

On the other hand, $q(s)$ can be written as

$$q(s) = [F_M(sI-A_M)^{-1}B_M + G_M] u_M(s) = \Phi(s) H_M(s) u_M(s) \tag{14}$$

Therefore, the overall TFM is $H_M(s)$—the desired model. This indicates that the model-following control based upon the exact model-matching method is achieved. Figure 1 shows the block diagram of this control system.

*Case II: Singular Control Matrix*

In order that $u_P(t)$ can be synthesized by Eqs. (12), the control matrix $G$ must be nonsingular. This requirement is known as the necessary and sufficient condition for system decoupling using state feedback as described in Ref. 1 where $d_j$, instead of $m_j - 1$, is used. The way of solving this problem is to augment the system using a suitable unimodular matrix $U_L(s)$.[6] In Ref. 6, it is stated that a unimodular matrix can always be found so that $U_L(s) C_{\mathrm{adj}}(sI-A)B$ is row proper.

Defining a stable scalar polynomial $\zeta(s)$ such that $U_L(s)/\zeta(s)$ is a proper TFM, the minimal realization of this TFM is found (not uniquely) to be

$$U_L(s)/\zeta(s) = C_C(sI-A_C)^{-1}B_C + D_C \tag{15}$$

If system (1) is augmented with Eq. (15) such that $[U_L(s)/\zeta(s)]H(s)$, the following new system is obtained:

$$\dot{x}_A(t) = A_A x_A(t) + B_A u_P(t) \tag{16a}$$

$$\eta_A(t) = C_A x_A(t) \tag{16b}$$

where

$$x_A^T(t) = [x_C^T(t),\ x^T(t)] \tag{17a}$$

$$\eta_A \in R^p \tag{17b}$$

$$x_C^T(0) = [0,\ldots,0] \tag{17c}$$

$$A_A = \begin{bmatrix} A_C & B_C C \\ 0 & A \end{bmatrix} \tag{17d}$$

$$B_A = \begin{bmatrix} 0 \\ B \end{bmatrix} \tag{17e}$$

$$C_A = [C_C,\ D_C C] \tag{17f}$$

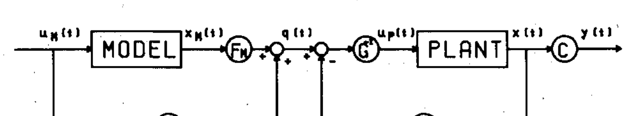


**Fig. 1 Block diagram of the controller (case I).**

It should be noted that the control matrix $G_A$ for system (16) is now nonsingular as its TFM is row proper. Similarly, augmenting model (3) with Eq. (15) gives

$$\dot{x}_B(t) = A_B x_B(t) + B_B u_M(t) \quad (18a)$$

$$\eta_B(t) = C_B x_B(t) \quad (18b)$$

where

$$x_B^T(t) = [x_D^T(t),\ x_M^T(t)] \quad (19a)$$

$$\eta_B \in R^p \quad (19b)$$

$$x_D^T(0) = [0,\ldots,0] \quad (19c)$$

$$A_B = \begin{bmatrix} A_C & B_C C_M \\ 0 & A_M \end{bmatrix} \quad (19d)$$

$$B_B = \begin{bmatrix} 0 \\ B_M \end{bmatrix} \quad (19e)$$

$$C_B = [C_C,\ D_C C_M] \quad (19f)$$

The control input can be obtained using exactly the same procedure as was used for Eqs. (12)

$$u_p(t) = G_A^{-1}[-F_A x_A(t) + q_A(t)] \quad (20a)$$

$$q_A(t) = F_B x_B(t) + G_B u_M(t) \quad (20b)$$

where $F_A, G_A$ and $F_B, G_B$ are defined for Eqs. (16) and (18) in the same way as for $F, G$ and $F_M, G_M$. The error equation in this case is

$$\Phi(s)[\eta_A(t) - \eta_B(t)] = [\Phi(s) U_L(S)/\zeta(s)] e(t) = 0 \quad (21)$$

The TFM is $[U_L(s)/\zeta(s)]^{-1}\Phi^{-1}(s)$ from $q_A(t)$ through $y(t)$ and $\Phi(s)[U_L(s)/\zeta(s)]H_M(s)$ from $u_M(t)$ to $q_A(t)$. Exact model-matching is thus achieved by the input [Eqs. (20)]. The controller structure is depicted in Fig. 2.

### Using Plant Inputs and Outputs

As system (1) is observable, it can be expressed by the following subsystems[5]:

$$\dot{x}_{Ti}(t) = \sum_{j=1}^{P} A_{ij} x_{Tj}(t) + B_i u_P(t)$$
$$= A_{0i} x_{Ti}(t) + \sum_{j=1}^{P} \alpha_{ij} x_{\sigma_j(t)} + B_i u_P(t), \quad (i = 1,2,\ldots p) \quad (22)$$

where

$$x_{Ti}^T(t) = [x_{\sigma i-1}(t),\quad x_{\sigma_{i-1}+2}(t),\ldots,\quad x_{\sigma_i}(t)] \quad (23a)$$

$$A = [A_{ij}] \quad (23b)$$

$$A_{ii} = \left[\begin{array}{c|c} 0..0 & \\ \hline I_{d_i-1} & \alpha_{ii} \end{array}\right] \quad (23c)$$

$$A_{ij} = [0_{d_i \times (d_i-1)} \,|\alpha_{ij}] \quad (23d)$$

$$A_{0i} = \left[\begin{array}{c|c} 0..0 & 0 \\ \hline I_{d_i-1} & \begin{matrix}\cdot \\ \cdot \\ 0\end{matrix} \end{array}\right] \quad (23e)$$

$$B^T = [B_1^T, B_2^T, \ldots, B_p^T] \quad (23f)$$

$$C = [0\ \psi_1\ 0\ \psi_2 \ldots 0\ \psi_p] \quad (23g)$$

$$\psi_i^T = [\underbrace{0\ \ldots\ 0}_{i}\ 1\ \underbrace{*\ldots *}_{P-i}] \quad (23h)$$

In Eq. (23h), the asterisk denotes a nonzero real variable, $d_i$ $(i = 1,2,\ldots,p)$ are observability indices, and

$$\sigma_j = \sum_{i=1}^{j} d_i \, (\sigma_0 = 1)$$

On the other hand, the output can be rewritten as

$$y(t) = \bar{C} x_\sigma(t) \quad (24)$$

where

$$x_\sigma^T(t) = [x_{\sigma 1}(t), x_{\sigma 2}(t), \ldots, x_{\sigma p}(t)] \in R^{1\times p} \quad (25a)$$

$$\bar{C} = [\psi_1, \psi_2, \ldots, \psi_p] \in R^{p\times p} \quad (25b)$$

Selecting the design parameters $\lambda_i$ $(i = 1,2,\ldots,d_i - 1)$ such that $\lambda_i \neq \lambda_j$ for $i \neq j$ and $\lambda_i > 0$, Eqs. (20) can be written as

$$\dot{z}_{Ti}(t) = J_i z_{T_i}(t) + K_i y(t) + L_i u_P(t) \quad (26)$$

where

$$x_{Ti}(t) = T_i(\lambda) z_{Ti}(t) \quad (27a)$$

$$x_{\sigma_i}(t) = z_{\sigma_{i-1}+1}(t) \quad (27b)$$

$$J_j = \left[\begin{array}{c|c} 0 & 1\ldots 1 \\ \cdot & \\ \cdot & \overline{\phantom{xx}} \\ 0 & \Lambda_i \end{array}\right] \quad (27c)$$

$$\Lambda_i = \mathrm{diag}(-\lambda_j), \qquad (j = 1,2,\ldots,d_i - 1) \quad (27d)$$

$$K_i = A_i \bar{C}^{-1} \quad (27e)$$

$$A_i = [a_{i1}, a_{i2}, \ldots, a_{ip}] \quad (27f)$$

$$\left[\begin{array}{c|c} 0..0 & \\ \hline I_{d_i-1} & \alpha_{ii} \end{array}\right] = T_i(\lambda) \left[\begin{array}{c|c} \alpha_{ii} & 1..1 \\ \hline & \Lambda_i \end{array}\right] T_i^{-1}(\lambda) \quad (27g)$$

$$\alpha_{ij} = T_i^{-1}(\lambda)\alpha_{ij} \quad (27h)$$

$$L_i = T_i^{-1}(\lambda) B_i \quad (27i)$$

$$T_i(\lambda) = \left[\begin{array}{c|c|c|c|c} C(\lambda_i) & C(\Lambda_i/\lambda_1) & C(\Lambda_i/\lambda_2) & \ldots & C(\Lambda_i/\lambda_{di} - 1) \\ & 0 & 0 & & 0 \end{array}\right] \quad (27j)$$

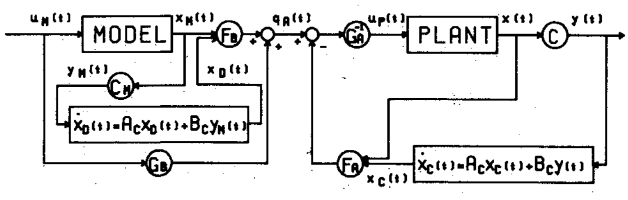


**Fig. 2 Block diagram of the controller (case II).**

In the above, $C(\lambda_i)$ is a vector containing the coefficients of the characteristic polynomial

$$\prod_{k=1}^{d_i} (z+\lambda_k)$$

of a diagonal matrix $\Lambda_i$, and $C(\Lambda_i/\lambda_j)$ is defined in the same way for

$$\prod_{\substack{k=1\\k\neq j}}^{d_i} (z+\lambda_k)$$

Now define the following state variable filters:

$$\dot{v}_i(t)=\Lambda_i v_i(t)+P_i y^T(t), \qquad v_i(0)=v_{i0} \tag{28a}$$

$$\dot{w}_i(t)=\Lambda_i w_i(t)+P_i u^T(t), \qquad w_i(0)=w_{i0} \tag{28b}$$

where

$$v_i(t)=[v_{i1}(t),v_{i2}(t),...,v_{ip}(t)]\in R^{(di-1)\times p} \tag{29a}$$

$$w_i(t)=[w_{i1}(t),w_2(t),...,w_{ip}(t)]\in R^{(di-1)\times p} \tag{29b}$$

$$P_i^T=[1,1,...,1]\in R^{1\times(d_i-1)} \tag{29c}$$

Using these filters, Eq. (27a) can be modified[5] to

$$x(t)=T(\lambda)D\mu(t) \tag{30}$$

where

$$\mu(t)=[y^T,v_{01}^T,...,v_{0p}^T,w_{01}^T,...,w_{0p}^T]^T\in P_i^{(2d_ip-p)} \tag{31a}$$

$$D_i=\left[\begin{array}{c|c|c} {}^1D & 0_{1\times p(d_0-1)} & 0_{1\times P(d_0-1)} \\ \hline 0_{(d_i-1)\times p} & {}^2D & {}^3D \end{array}\right]$$

$${}^1D=[(C^{-1})_{i1}(C^{-1})_{i2}...(C^{-1})_{ip}]\in R^{1\times p}$$

$${}^2D=[{}^2D_i...{}^2D_p],\ {}^3D=[{}^3D_1...{}^3D_p] \tag{31b}$$

$${}^2D_j=\left[\begin{array}{ccc|c} \bar{K}_{i1j} & \cdots & 0 & \\ \vdots & \cdots & \vdots & 0_{(d_i-1)(d_0-d_i)} \\ 0 & \cdots & \bar{K}_{i(d_i-1)j} & \end{array}\right]\in R^{(d_i-1)(d_0-1)}$$

$${}^3D_j=\left[\begin{array}{ccc|c} \bar{L}_{i1j} & \cdots & 0 & \\ \vdots & \cdots & \vdots & 0_{(d_i-1)(d_0-d_i)} \\ 0 & \cdots & \bar{L}_{i(d_i-1)1} & \end{array}\right]\in R^{(d_i-1)(d_0-1)}$$

$$T(\lambda)=\text{block}[T_i(\lambda)], \qquad (i=1,2,...,p) \tag{31c}$$

$$D^T=[D_1^T,D_2^T,...,D_p^T] \tag{31d}$$

and $\bar{K}_i$ and $\bar{L}_i$ denote the matrices made by deleting the first columns of $K_i$ and $L_i$, respectively, $d_0$ is an observability index (i.e., max. $d_i$), and $v_{0i}$ and $w_{0i}$ defined for $d_0$ as in Eqs. (29a) and (29b).

The control inputs of Eqs. (12) and (20) are now modified using Eq. (30), so they can be generated without the explicit use of plant state variables, as

$$u_P(t)=G^{-1}[-FT(\lambda)D\mu(t)+F_Mx_M(t)+G_Mu_M(t)] \tag{32a}$$

$$u_P(t)=G_A^{-1}\left[-F_A\begin{bmatrix} x_C(t) \\ T(\lambda)D\mu(t)\end{bmatrix}+F_Mx_M(t)+G_Mu_M(t)\right] \tag{32b}$$

## Decoupled CCV Flight Control

The longitudinal dynamics of the aircraft considered are given by the following linearized time-invariant equations

$$\dot{u}(t)=X_uu(t)+X_ww(t)-g\theta(t)$$

$$\dot{w}(t)=Z_uu(t)+Z_ww(t)+U_0q(t)+Z_{\delta_f}\delta_f(t)+Z_{\delta_e}\delta_e(t)$$

$$\dot{q}(t)=(M_u+M_{\dot{w}}Z_u)u(t)+(M_w+M_{\dot{w}}Z_w)w(t)$$
$$+(M_q+U_0M_{\dot{w}})q(t)+(M_{\dot{w}}Z_{\delta_f}+M_{\delta_f})\delta_f(t)$$
$$+(M_{\dot{w}}Z_{\delta_e}+M_{\delta_e})\delta_e(t)$$

$$a_z(t)=\dot{w}(t)-U_0q(t),\ n_z(t)=-a_z(t)/g,\ \dot{\theta}(t)=q(t) \tag{33}$$

where $u(t)$ and $w(t)$ are velocity increments in the forward and vertical directions, $\theta(t)$ is the pitch angle, $q(t)$ the pitch rate, $\delta_e(t)$ and $\delta_f(t)$ deflections of elevator and flaperon, $a_z(t)$ the normal acceleration, $n_z(t)$ the load factor, and $X$ and $Z$ are stability and control derivatives. In the following design, the parameters of a hypothetical Japanese T-2 CCV at the condition of $H=20{,}000$ ft, Mach$=0.8$, $S=21.17$ m$^2$, and $I_y=9740$ kg-m$^2$ are used.

### Mode $\alpha_1$ with Nonsingular Control Matrix

In this mode, a constant flight-path angle must be maintained. Considering the following state variables, inputs and outputs

$$x(t)=[h(t),U_0\gamma(t),w(t),\theta(t),q(t)]^T \tag{34a}$$

$$y(t)=[h(t),\theta(t)]^T \tag{34b}$$

$$u_P(t)=[\delta_f(t),\delta_e(t)]^T \tag{34c}$$

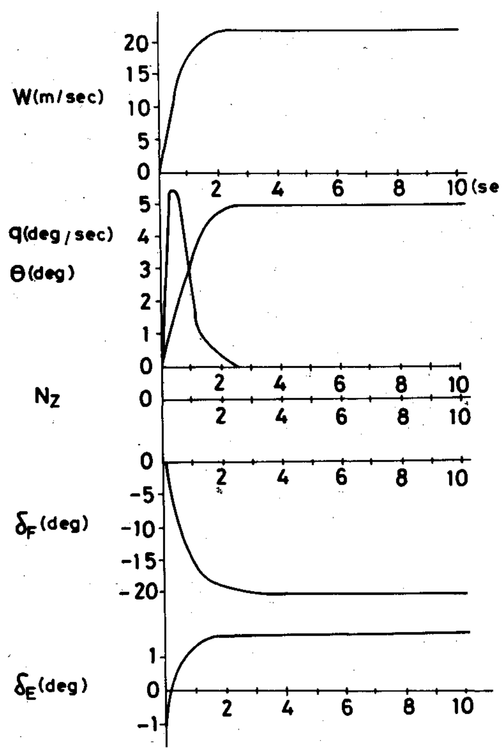


Fig. 3 Simulation results for mode $\alpha_1$.

can be obtained early. In terms of four constants $C_1,\ldots, C_4$

$$A=\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0.882 & 0 & 0 \\ 0 & 0 & -0.882 & 0 & 254.4 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -0.008 & 0 & -1.217 \end{bmatrix}, \quad B=\begin{bmatrix} 0 & 0 \\ 60.39 & 47.39 \\ -60.39 & -47.39 \\ 0 & 0 \\ -2.34 & -22.06 \end{bmatrix}, \quad C=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{35a-c}$$

The model is given by

$$A_M=\begin{bmatrix} 0 & 1 & 0 & 0 \\ -9 & -6 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -9 & -6 \end{bmatrix}, \quad B_M=\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad C_M=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{36}$$

The control matrix $G$ of the plant [Eqs. (35)] is obtained as

$$G=\begin{bmatrix} C_1AB \\ C_2AB \end{bmatrix}=\begin{bmatrix} 60.39 & 47.39 \\ -2.34 & -22.06 \end{bmatrix}$$

where $m_1=m_2=2$. As $|G|\neq 0$ in this case, the design of case 1 can be used and the control input according to Eqs. (12) is obtained

$$u_P(t)=\begin{bmatrix} -0.163 & -0.108 & -0.016 & -0.349 & -0.185 \\ 0.017 & 0.012 & 0.001 & 0.445 & 0.263 \end{bmatrix} x(t)+\begin{bmatrix} 0.018 & 0.039 \\ -0.002 & -0.049 \end{bmatrix} u_M(t) \tag{37}$$

where $f^1(s)=f^2(s)=s^2+6s+9$.

The control input in accordance with Eq. (32a) can be obained by replacing $x(t)$ in Eq. (37) with the following

$$x(t)=\begin{bmatrix} 2 & 2 & 1 & 0 & 0 \\ 3 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.0882 & 0 & -224.381 & 0 & -60.39 & 0 & -47.39 & 0 \\ 0 & 0 & 0 & 3.528 & 0 & 448.762 & 0 & 120.78 & 0 & 94.78 \\ 0.009 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.217 & 0 & -1.792 & 0 & -21.63 & 0 \end{bmatrix}\mu(t) \tag{38a}$$

where $\lambda_1$ and $\lambda_2$ were selected as 1 and 2, and

$$\dot{v}_0(t)=\begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} v_0(t)+\begin{bmatrix} 1 \\ 1 \end{bmatrix} y^T(t)\ \epsilon R^{2\times 2} \tag{38b}$$

$$\dot{w}_0(t)=\begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} w_0(t)+\begin{bmatrix} 1 \\ 1 \end{bmatrix} u^T(t)\ \epsilon R^{2\times 2} \tag{38c}$$

To perform mode $\alpha_1$ with $\theta(t)=5$ deg and $h(t)=0$, the reference input was chosen as $u_M(t)=[0,0.785]$ and the simulation results are shown in Fig. 3. The response obtained using Eq. (38a) is the same as that by the input Eq. (37).

### Mode $\alpha_2$ with Singular Control Matrix

This mode is concerned with the control of vertical velocity at a constant pitch. Consider the following selection of the states, inputs, and outputs.

$$x(t)=[u(t),w(t),\theta(t),q(t)]^T \tag{39a}$$

$$y(t)=[w(t)+\theta(t),w(t)]^T \tag{39b}$$

$$u_P(t)=[\delta_f(t),\delta_e(t)]^T \tag{39c}$$

We now have

$$A=\begin{bmatrix} -0.0084 & 0.0385 & -9.8 & 0 \\ -0.077 & -0.882 & 0 & 254.4 \\ 0 & 0 & 0 & 1 \\ 0.0001 & -0.008 & 0 & -1.217 \end{bmatrix} \tag{40a}$$

$$B=\begin{bmatrix} 0 & 0 \\ -60.39 & -47.394 \\ 0 & 0 \\ -2.34 & -22.062 \end{bmatrix} \tag{40b}$$

$$C=\begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{40c}$$

As the control matrix $G$ of this system is

$$G=\begin{bmatrix} C_1B \\ C_2B \end{bmatrix}=\begin{bmatrix} -60.39 & -47.39 \\ -60.39 & -47.39 \end{bmatrix};\ \text{singular}$$

where $m_1=m_2=1$, a unimodular matrix $U_L(s)$ was chosen to be

$$U_L(s)=\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \tag{41a}$$

so that

$$G_A=\begin{bmatrix} -60.39 & -47.39 \\ 2.34 & 22.06 \end{bmatrix} \tag{41b}$$

which is nonsingular. Note that $m_2$ is increased from 1 to 2. The model selected is

$$y_m(t)=\text{diag}[51.84/(s^2+12.96s+51.84)]u_M(t) \tag{42}$$

The control input by Eqs. (12) is

$$u_P(t)=\begin{bmatrix} -0.001 & 0.175 & -3.880 & 3.866 \\ 0 & -0.018 & 4.944 & 0.441 \end{bmatrix}x(t)$$
$$+\begin{bmatrix} 96.88 & 13.22 & -96.88 & -14.16 \\ -123.45 & -17.95 & 123.45 & 18.05 \end{bmatrix}x_M(t)$$
$$+\begin{bmatrix} 2.01 & -2.01 \\ -2.56 & 2.56 \end{bmatrix}u_M(t) \tag{43}$$

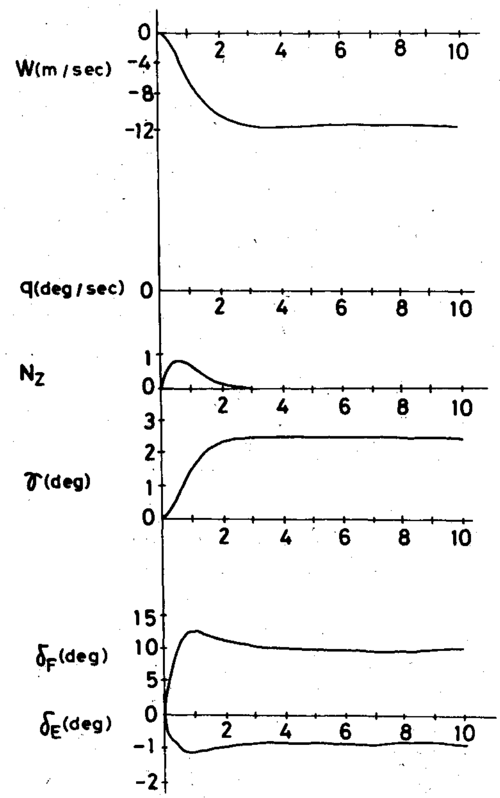


**Fig. 4 Simulation results for mode $\alpha_2$.**

where $\lambda_1$ was chosen to be 2 and

$$\dot{v}_0(t)=-2v_0(t)+y^T(t)\ \epsilon R^{1\times 2} \tag{44b}$$

$$\dot{w}_0(t)=-2w_0(t)+u^T(t)\ \epsilon R^{1\times 2} \tag{44c}$$

The computational results are given in Fig. 4 for the reference input $u_M(t)=[10,10]^T$. Both control inputs, i.e., Eq. (43) and the one using Eq. (44a), give the same responses.

## Conclusions

A new model-following controller composed of state feedback and input dynamics compensators was described and applied to a decoupled flight control. This design is applicable to systems which have singular control matrices. Simulation studies showed that the design is effective. Although the decoupled control of a control-configured-vehicle-type aircraft is mainly studied in this paper, the proposed scheme can be used as well for nondecoupled control; this depends upon the selection of the model. The proposed methods can be applied to the other modes, including lateral motions, in exactly the same manner.

where $f^1(s)=s+10$ and $f^2(s)=s^2+20s+100$. Replacing $x(t)$ in the above equation with Eq. (30) gives the input by Eq.(32b). In this case,

$$x(t)=\begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}\times\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2.35 & -3.10 & -497.40 & -555.46 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.01 & 0.01 & 2.42 & 22.12 \end{bmatrix}\mu(t) \tag{44a}$$

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